The use of *Mathematica* for the classification of some nilpotent Lie algebras

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Abstract

The classification of nilpotent Lie algebras is nowadays an open problem [5]. In the last years have been studied the nilpotent Lie algebras leading to their classification up to dimension 7, as a particular type of the above, the classification of the filiform Lie algebras is giving up to dimension 11 and the classification of the 2-abelian filiform Lie algebras for the case of generic dimension. Cabezas and Gómez generalize the notion of filiform algebra to p-filiform algebra [4], [3], [2] (whose Goze's invariant is $(dim(\mathfrak{g})-p, 1, 1, \ldots, 1)$), also give the classification for the integer values of p between $dim(\mathfrak{g})-4$ and $dim(\mathfrak{g})-2$, for $p = dim(\mathfrak{g})-5$ we have classified the $(n-5)$-filiform Lie algebras with maximal derived algebra, where $n = dim(\mathfrak{g})$, [1].

The determination of Lie algebras is a difficult work, therefore we have developed some new methods which allows the classification of nilpotent Lie algebras.

In this paper, we show a program by using the *Mathematica* 3.0 package which allows the classification of the sets of laws of $(n-6)$-filiform Lie algebras of dimension 8.

1 Description of the program

Classifications problems up to isomorphisms can be reduced to determinate the structure constants of non-isomorphic algebras. As the structure constants depend on the basis, it is convenient to find a basis for which one has a maximum number of structure constants equal zero. We consider adapted basis.

Let be a family of $(n-6)$-filiform Lie algebras of dimension 8. For the classification of this family, it is necessary to find classic invariants. This is not always

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possible. Then, it is necessary to make the generic change of basis. This program allows to generate the new vectors and to compute new structure constants. Later we have to do a discussion over the output of program to can classify.

The way to find non-isomorphic algebras of dimension 8 can be summarized in the following steps

Step 1: We introduce the family, the conditions of bilinear and anti-symmetry and the condition of $(n-6)$-filiform Lie Algebras.

Step 2: We generate particular new adapted basis, where the structure constants can be zero.

Step 3: We compute the conditions to maintain some null bracket products. We select only most simple null bracket products.

Step 4: We obtain new structure constants of the family in the new adapted basis. In this way, we compute some bracket products.

Step 5: We check that the remain of null bracket products. In the new adapted basis these products have to be null.

Finally, we discuss if the nullities of new structure constants are invariants or zero. This discussion allows to classify, since it separates in non-isomorphic algebras.

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